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A model of heat transfer between fluidized beds and immersed surfaces

PEISHI CHEN and DAVID C. T. PEI*

Department of Chemical Engineering, University of Waterloo, Ontario, Canada N2L 3G1

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Abstract—In fully developed fluidized beds, the particle motion plays an over-riding important role in heat transfer process. In consideration of this fact, a new model is developed to describe the heat transfer between fluidized beds and immersed surfaces, based on the definition of a two-phase thermal boundary layer around the surface. The subsequent correlations of the maximum heat transfer coefficient and the optimum flow-rate are obtained. They agree well over a wide range of conditions with the data reported in the literature.

1. INTRODUCTION

THE KNOWLEDGE of the heat transfer between fluidized beds and immersed surfaces is of importance in the development of the fluidized bed boilers and various chemical processes. The heat transfer coefficient can be considered to consist of three additive components.

(1) Particle convective heat transfer;
(2) interstitial gas convective heat transfer;
(3) radiant heat transfer.

The third component becomes significant only at very high operating temperatures. In fully developed fluidized beds, the motion of the particles plays an over-riding role in the heat transfer process. This is because the solid particles in a fluidized bed usually have large heat capacity and high thermal conductivity. When the particles move close to the heating surface, they receive thermal energy, acting as a local heat sink. It is a well-known fact that, as the gas velocity increases beyond the minimum fluidization velocity, there is a rapid rise in heat transfer coefficient. A maximum heat transfer rate is reached at a certain optimum gas velocity, then reduces with further increase in gas velocity. This can be simply explained as a result of the vigorous motion of particles which intensifies the heat transfer process and the increasing bed voidage which reduces the heat transfer coefficient.

Various models have been proposed to describe the fluidized bed heat transfer process. Generally, three basic theories may be distinguished. The first kind of models are based on the fluid film theory. It is assumed that, the thermal resistance of the fluid film is affected by the velocity and the properties of the fluid as well as the intensity of the particle movement. As this kind of model takes no account of the influence of particle properties, the subsequent formulae are usually not in good agreement with the experimental results. The second kind of models are based on the transient heat conduction between the immersed surface and single particles, some models include the thermal resistance of the fluid layer at the surface, such as the models proposed by Ziegler [1], Botterill [2], Yamazaki[3] and Wick and Fetting [4]. In these models, allowance is made for the influence of particle properties. However, the foregoing formulae can only apply to a particulate fluidized bed, but not a bubbling system. For a bubbling system, Mickley and Fairbanks [5] developed a 'packet model', based on the surface renewal theory. According to this model, the heat transfer is performed by the moving 'packets'; and the heat transfer rate depends on the replacing frequency and the thermal properties of the 'packets'. Baskakov [6] modified this model by including the effective gap resistance between the packet and surface as well as the gas convective heat transfer. Generally, those models were not successful for the predictive purpose of various experimental results, as reviewed by Saxena *et al.* [7] and Botterill *et al.* [8]. On the other hand, many empirical correlations were proposed based on experimental data within narrow ranges of particle Archimedes number and usually lack of fundamental analysis of the heat transfer between fluidized beds and immersed surfaces.

Saxena *et al.* [7] have suggested that the particle volumetric heat capacity, the gas thermal conductivity and the particle residence time are important factors to develop a generalized correlation. Toomey and Johnstone [9] observed that, even at high gas flow-rates, the fine particles still remain at the surface. Koppel *et al.* [10] measured the frequency of particle exchange between a surface and the bulk of the fluidized bed and found that the relative spread in residence time was wide when operating at high gas flow-rates. Ishida *et al.* [11] measured the local particle velocity in a fluidized bed. These direct measurements make it possible to properly construct a heat transfer model of fluidized beds. Consequently, a new model of the heat transfer between fluidized beds and immersed surfaces is developed. The subsequent correlations are derived for predicting the maximum heat transfer coefficient and optimum gas flow-rate. These correlations agree well

*To whom correspondence should be addressed.

NOMENCLATURE

a	parameter, equation (14), dimensionless
Ar	Archimedes number, $d_p^3\rho_f(\rho_s-\rho_f)g/\mu^2$, dimensionless
c	constant, equation (8)
c_{pf}, c_{ps}	specific heat of fluid and particle [J kg^{-1} K^{-1}]
c_{pM}	specific heat of boundary layer [J kg^{-1} K^{-1}]
d_p	diameter of particle [m]
f_b	fraction of surface exposed to bubbles, equation (10), dimensionless
h, h_{max}	heat transfer coefficient and its maximum value [W m^{-2} K^{-1}]
k_f, k_s	thermal conductivity of fluid and particle [W m^{-1} K^{-1}]
k_M	thermal conductivity of boundary layer [W m^{-1} K^{-1}]
$\mathscr{L}$	Laplace transform
m	constant, equation (15), dimensionless
Nu, Nu_{max}	Nusselt number and its maximum value, dimensionless
n	constant, equation (19), dimensionless
n_W	packet replacing frequency, equation (11) [s^{-1}]
Pr, Pr_M	Prandtl number, $\mu c_p/k$, of fluid and boundary layer, dimensionless
R	radius of immersed surface [m]
Re, Re_{opt}	particle Reynolds number, $d_p u_0 \rho_f/\mu$, and optimum Reynolds number, dimensionless
Re_{mf}	Reynolds number for minimum fluidization, dimensionless
s	renewal frequency, equation (2) [s^{-1}]
u_0, u_{mf}	superficial and minimum fluidization velocity [m s^{-1}]
$\lvert\bar{u}_s\rvert, \bar{u}_b$	mean particle and bubble rising velocity [m s^{-1}].

Greek symbols

α_M	thermal diffusivity of boundary layer [m^2 s^{-1}]
δ, δ_T	thickness of momentum and thermal boundary layer [m]
$\varepsilon, \varepsilon_{mf}$	bed voidage and bed voidage at minimum fluidization, dimensionless
ε_{opt}	bed voidage at the optimum flowrate, dimensionless
μ, μ_M	viscosity of fluid and boundary layer [kg m^{-1} s^{-1}]
ν, ν_M	kinematic viscosity of fluid and boundary layer [m^2 s^{-1}]
ρ_f, ρ_s	density of fluid and particles [kg m^{-3}]
ρ_M	density of boundary layer, [kg m^{-3}]
τ	integral parameter, equation (4).

with the experimental data reported in the literature over a wide range of operating conditions.

2. THEORETICAL MODEL

In a fully developed fluidized bed, the immersed surface is surrounded by the fluid and moving particles. These particles receive heat at the heating surfaces and quickly transfer the heat to the fluid and the particles around the heating surface. This transfer process may cause the vanishment of the radial temperature gradient at a certain distance away from the heating surfaces. Hence, a thermal boundary layer can be defined based on this analysis. It is also evident that, this thermal boundary layer is from time to time renewed by the moving particles and fluidizing fluid. Moreover, it is important to realize the fact that the two-phase thermal boundary layer may have different thickness along the immersed surface. However, in fully developed fluidized beds, an average thickness of the boundary layer can be assumed to deal with the total heat transfer coefficient. The geometry of this model is shown in Fig. 1. The following assumptions are made:

(1) a spherical or cylindrical heating surface is immersed in a fluidized bed, with a radius of R and average temperature of T_1;

(2) there is a concentric boundary layer with an average thickness of δ_T surrounding the heating surface, which consists of both solid particles and fluid;

(3) at the outer boundary of the layer, the radial temperature derivative is zero;

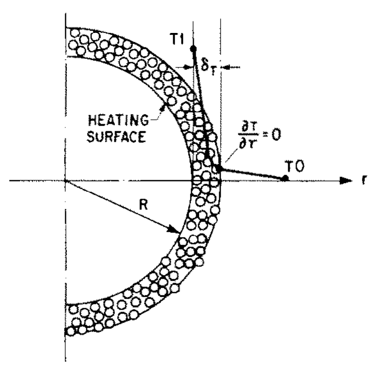

FIG. 1. Geometry of the model.

(4) the thermo-physical properties of both fluid and particles are constant;

(5) radiant heat transfer is negligible.

Now, one may formulate this problem using the method of penetration theory developed by Danckwerts [12]. At time $t = 0$, the boundary layer is at an average temperature T_0. It is heated up by the immersed surface and renewed by the moving particles and the fluidizing fluid simultaneously. The transient heat transfer problem can be described by the following differential equation and boundary conditions,

$$\nabla^2 T = \frac{1}{\alpha_M}\frac{\partial T}{\partial t} \quad R < r < R+\delta_T \tag{1}$$

$$T(R,t) = T_1 \tag{1a}$$

$$\frac{\partial T}{\partial r}(R+\delta_T, t) = 0 \tag{1b}$$

$$T(r,0) = T_0. \tag{1c}$$

By taking the Laplace transform of equation (1) with respect to time and solving this equation, the average temperature distribution can be obtained, (i) for immersed spherical surface,

$$\frac{\bar{T}-T_0}{T_1-T_0} = \frac{R}{r}\left\{\frac{\left((R+\delta_T)\sqrt{\frac{s}{\alpha_M}} - \tanh\left[(R+\delta_T-r)\sqrt{\frac{s}{\alpha_M}}\right]\right)\cosh\left[(R+\delta_T-r)\sqrt{\frac{s}{\alpha_M}}\right]}{\left((R+\delta_T)\sqrt{\frac{s}{\alpha_M}} - \tanh\left[\delta_T\sqrt{\frac{s}{\alpha_M}}\right]\right)\cosh\left[\delta_T\sqrt{\frac{s}{\alpha_M}}\right]}\right\} \tag{2}$$

and (ii) for immersed cylindrical surface,

$$\frac{\bar{T}-T_0}{T_1-T_0} = \frac{I_0\left(r\sqrt{\frac{s}{\alpha_M}}\right)K_1\left[(\delta_T+R)\sqrt{\frac{s}{\alpha_M}}\right] + K_0\left(r\sqrt{\frac{s}{\alpha_M}}\right)I_1\left[(\delta_T+R)\sqrt{\frac{s}{\alpha_M}}\right]}{I_0\left(R\sqrt{\frac{s}{\alpha_M}}\right)K_1\left[(\delta_T+R)\sqrt{\frac{s}{\alpha_M}}\right] + K_0\left(R\sqrt{\frac{s}{\alpha_M}}\right)I_1\left[(\delta_T+R)\sqrt{\frac{s}{\alpha_M}}\right]} \tag{3}$$

where

$$\bar{T} = \int_0^\infty s\,e^{-s\tau}\,T(r,\tau)\,d\tau = s\mathscr{L}(T) \tag{4}$$

and s is defined as the renewal frequency of the surface layer. By using Danckwerts' approach [12], the heat transfer coefficient can be obtained in the form of Nusselt number, for immersed spherical surface,

$$Nu = \frac{k_M}{k_f}\left\{\frac{d_p}{R} + \frac{d_p\left[(R+\delta_T)\frac{s}{\alpha_M}\tanh\left(\delta_T\sqrt{\frac{s}{\alpha_M}}\right) - \sqrt{\frac{s}{\alpha_M}}\right]}{\left[(R+\delta_T)\sqrt{\frac{s}{\alpha_M}} - \tanh\left(\delta_T\sqrt{\frac{s}{\alpha_M}}\right)\right]}\right\} \tag{5}$$

for immersed cylindrical surface,

$$Nu = \frac{k_M}{k_f}d_p\sqrt{\frac{s}{\alpha_M}}\left\{\frac{I_1\left(R\sqrt{\frac{s}{\alpha_M}}\right)K_1\left[(R+\delta_T)\sqrt{\frac{s}{\alpha_M}}\right] + K_1\left(R\sqrt{\frac{s}{\alpha_M}}\right)I_1\left[(R+\delta_T)\sqrt{\frac{s}{\alpha_M}}\right]}{I_0\left(R\sqrt{\frac{s}{\alpha_M}}\right)K_1\left[(R+\delta_T)\sqrt{\frac{s}{\alpha_M}}\right] + K_0\left(R\sqrt{\frac{s}{\alpha_M}}\right)I_1\left[(R+\delta_T)\sqrt{\frac{s}{\alpha_M}}\right]}\right\}. \tag{6}$$

If the fluidized bed is fully developed, the particle velocity is large enough and $R \gg d_p$, equations (5) and (6) can be simplified as,

$$Nu = \frac{k_M}{k_f}d_p\sqrt{\frac{s}{\alpha_M}}. \tag{7}$$

Usually, the renewal frequency of the surface layer s is assumed to be proportional to the ratio of the characteristic velocity to the characteristic length [13]. When the particle convective heat transfer dominates, the mean particle velocity $|\bar{u}_s|$ and the particle diameter are the characteristic velocity and length, therefore

$$s = c^2\frac{|\bar{u}_s|}{d_p}\frac{\delta_T}{\delta} = c^2\frac{|\bar{u}_s|}{d_p}Pr_M^{-1/3}. \tag{8}$$

Here the effect of the thermal boundary-layer thickness is also considered. From the boundary-layer theory [14], the ratio of the thermal boundary-layer thickness to that of the momentum boundary layer can be taken to be the reciprocal of the cubic root of Prandtl number, i.e. $\delta_T/\delta = Pr_M^{-1/3}$, and Pr_M is the Prandtl number of the two-phase boundary layer.

3. SPECIAL ANALYSIS

Some interesting observations are noted from the above solution. Provided Pr_M is constant for a given system and considering the heat capacity of the two-phase boundary layer is approximately equal to that of the solid phase, for bubbling beds,

$$\rho_M c_{pM} = \rho_s c_{ps}(1-\varepsilon) = \rho_s c_{Ps}(1-\varepsilon_{mf})(1-f_b). \quad (9)$$

Combining equation (7) with equation (9), it yields

$$Nu = c\frac{d_p}{k_f}\sqrt{\frac{|\bar{u}_s|}{d_p}\rho_s(1-f_b)(1-\varepsilon_{mf})k_M c_{Ps}} \quad (10)$$

which is similar to Mickley's correlation [5],

$$Nu = 1.13\frac{d_p}{k_f}\sqrt{n_w\rho_s(1-f_b)(1-\varepsilon_{mf})k_M c_{Ps}} \quad (11)$$

where n_w is the replacing frequency of the 'packets', as defined by Mickley.

If the effect of Pr_M is taken into account, equation (7) becomes

$$Nu = \text{const}\cdot\left(\frac{k_M}{k_f}\right)^{2/3}\left[\frac{|\bar{u}_s|d_p\rho_s(1-\varepsilon)}{\mu}\right]^{1/2} \times\left(\frac{\mu c_{Ps}}{k_f}\right)^{1/3}\left(\frac{\mu}{\mu_M}\right)^{1/6}. \quad (12)$$

From Maxwell's theory [15], the thermal conductivity of a heterogeneous material can be predicted by the following equation,

$$k_M = k_f\left[\frac{1-(1-\varepsilon)\left(1-a\frac{k_s}{k_f}\right)}{1+(1-\varepsilon)(a-1)}\right] \quad (13)$$

where

$$a = \frac{3k_f}{2k_f+k_s}. \quad (14)$$

Abrahamsen and Geldart [16] studied the effective thermal conductivity of the dense phase in fluidized beds based on the result of Maxwell. They found that the effective thermal conductivity of the dense phase was not sensitive to the gas velocity and the bed voidage, but the thermal conductivity of fluid. It can be interpreted as, in most cases, the thermal conductivity of solid is much larger than that of gas, i.e. $k_s \gg k_f$ and the voidage of the boundary layer is not sensitive to the gas velocity. According to their experimental results, the correlation may be expressed as

$$k_M = m\cdot k_f \quad (15)$$

where m can be taken as a constant.

In equation (12), μ, μ_M are the viscosities of the fluid and the two-phase boundary layer respectively. Since relatively few quantitative viscosity data for fluidized beds are published, it is hard to correlate the viscosity of fluidized beds based on these data. Schügerl [17] studied the viscosity of fluidized beds. Their experimental results show that the apparent kinematic viscosity of a fully developed fluidized bed is independent of particle diameter and appears to be proportional to the particle density, i.e.

$$\frac{\nu}{\nu_M} \propto \frac{\rho_f}{\rho_s}.$$

Therefore, equation (12) may be simplified as

$$Nu = \text{const.}\left[\frac{|\bar{u}_s|d_p\rho_f}{\mu}\right]^{1/2}(1-\varepsilon)^{1/3}Pr^{1/3}\left(\frac{\rho_s}{\rho_f}\right)^{1/6}\left(\frac{c_{Ps}}{c_{Pf}}\right)^{1/3}. \quad (16)$$

Mcguigan and Elliot [18] also studied the viscosity of fluidized beds. According to their experimental results, the viscosity of fluidized beds appears to be proportional to the particle density, i.e.

$$\frac{\mu}{\mu_M} \propto \frac{\rho_f}{\rho_s}.$$

Equation (12) may subsequently have the following form,

$$Nu = \text{const.}\left[\frac{|\bar{u}_s|d_p\rho_f(1-\varepsilon)}{\mu}\right]^{1/2}Pr^{1/3}\left(\frac{\rho_s c_{Ps}}{\rho_f c_{Pf}}\right)^{1/3}. \quad (17)$$

Ishida *et al.* [11] measured the particle velocity in a high temperature fluidized bed by using quartz fibre optic sensors. As shown in Fig. 2, their results indicate that the mean particle velocity $|\bar{u}_s|$ is a function of $(u_0 - u_{mf})$, and $|\bar{u}_s|$ approaches $\bar{u}_b$ when $(u_0 - u_{mf})$ is about $0.5\bar{u}_b$ for fine particles. Since the maximum heat transfer is directly resulted from the vigorous particle motion and the increasing bed voidage, the correlation of the optimum flow-rate may have the form $(Re_{opt} - Re_{mf}) = f(Ar)$. If the literature data [4, 19–26] are plotted in such a manner, the following correlation can be derived, as shown in Fig. 3.

$$Re_{opt} - Re_{mf} = 0.215\,Ar^{0.4}$$

$$20 < Ar < 2\times10^4 \quad (18a)$$

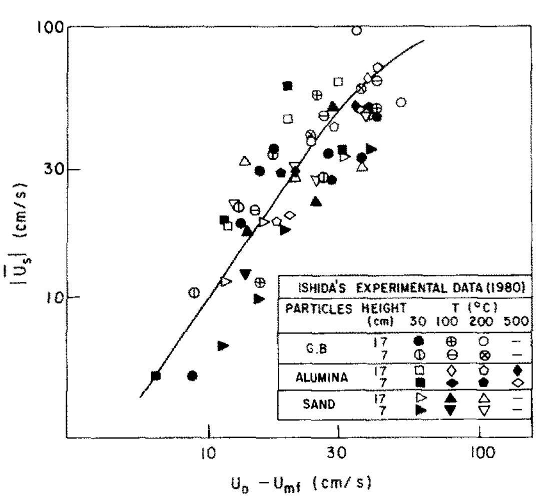

FIG. 2. $|\bar{u}_s|$ vs $u_0 - u_{mf}$, ref. [11].

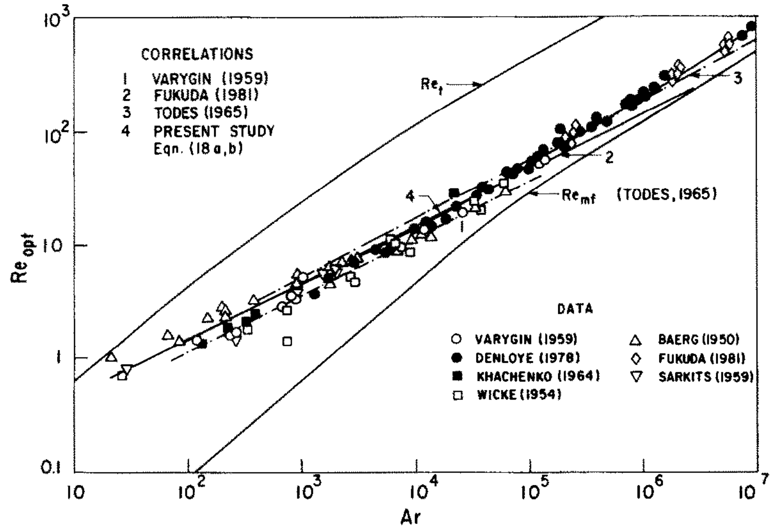

FIG. 3. Correlation of Re_{opt}.

$$Re_{opt} - Re_{mf} = 0.060\,Ar^{0.52}$$

$$2 \times 10^4 < Ar < 10^7. \quad (18b)$$

Moreover, the voidage of a fluidized bed is related to the ratio of the superficial velocity to the particle terminal velocity, which is a function of particle Archimedes number. It is expected that, at the optimum flow-rate, the bed voidage may also be a function of particle Archimedes number. Consequently, the correlation of maximum heat transfer coefficient based on equation (16) may have the form

$$Nu_{max} = \text{const.}\,Ar^n\,Pr^{1/3}\left(\frac{\rho_s}{\rho_f}\right)^{1/6}\left(\frac{c_{ps}}{c_{pf}}\right)^{1/3} \quad (19)$$

The values of

$$Nu_{max}Pr^{-1/3}\left(\frac{\rho_s}{\rho_f}\right)^{-1/6}\left(\frac{c_{Ps}}{c_{Pf}}\right)^{-1/3}$$

from the literature data [4, 6, 20–24, 26–29] are plotted vs particle Archimedes number as shown in Fig. 4. A general correlation is obtained, with a standard deviation of 15%,

$$Nu_{max} = 0.28\,Ar^{0.2}\,Pr^{1/3}\left(\frac{\rho_s}{\rho_f}\right)^{1/6}\left(\frac{c_{Ps}}{c_{Pf}}\right)^{1/3}$$

$$20 < Ar < 2 \times 10^4 \quad (20a)$$

$$Nu_{max} = 0.049\,Ar^{0.37}\,Pr^{1/3}\left(\frac{\rho_s}{\rho_f}\right)^{1/6}\left(\frac{c_{Ps}}{c_{Pf}}\right)^{1/3}$$

$$2 \times 10^4 < Ar < 10^7. \quad (20b)$$

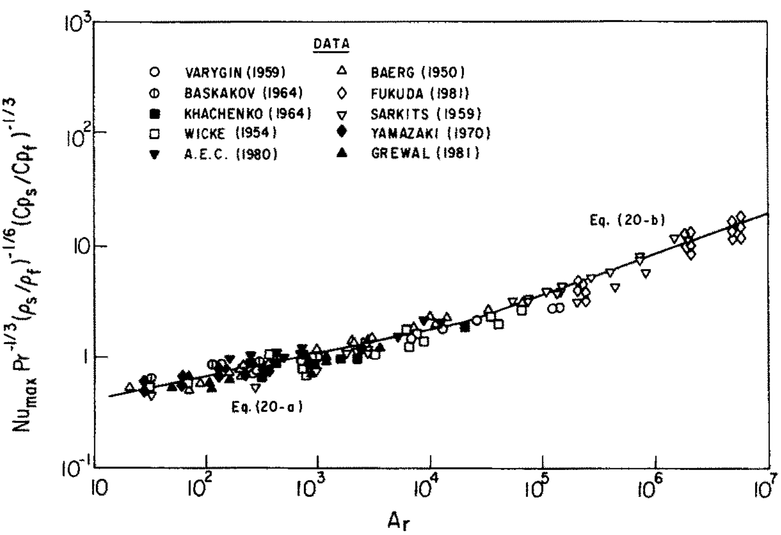

FIG. 4. Correlation of Nu_{max}, equation (20a,b).

From equation (17), the literature data of maximum heat transfer coefficient may be correlated as

$$Nu_{max} = \text{const.}\, Ar^n\, Pr^{1/3} \left(\frac{\rho_s c_{Ps}}{\rho_f c_{Pf}}\right)^{1/3}. \quad (21)$$

A similar correlation is obtained, as shown in Fig. 5, with a standard deviation of 18%,

$$Nu_{max} = 0.074\, Ar^{0.20}\, Pr^{1/3} \left(\frac{\rho_s c_{Ps}}{\rho_f c_{Pf}}\right)^{1/3} \quad 20 < Ar < 2\times10^4 \quad (22a)$$

$$Nu_{max} = 0.013\, Ar^{0.37} Pr^{1/3} \left(\frac{\rho_s c_{Ps}}{\rho_f c_{Pf}}\right)^{1/3} \quad 2\times10^4 < A_r < 10^7. \quad (22b)$$

Table 1. Comparison of prediction deviations between different correlations

Immersed surfaces in experiments	Tube (%)	Sphere (%)
S.D. of Nu_{max} Prediction		
Zabrosky *et al.* [30]	27.0	13.7
Grewal [27]	19.5	12.4
Equation (20a,b)	16.7	9.0
Equation (22a,b)	22.6	12.6
Maximum deviation of Nu_{max} prediction		
Zabrosky *et al.* [30]	94.0	31.5
Grewal [27]	38.4	32.0
Equation (20a,b)	37.2	27.9
Equation (22a,b)	56.1	39.1

4. DISCUSSION

For the first time, as shown in Figs. 3–5, this new model provides a sound theoretical analysis to the correlations on the optimum gas velocity and the maximum heat transfer coefficient between fluidized beds and immersed surfaces. Predictions of the present correlations were compared with that of some empirical correlations [27, 30] in Table 1. More than 100 experimental data in the range of $20 < Ar < 2\times10^4$ corresponding to different experimental conditions were used in comparison. It shows that equation (20a,b) gives best prediction with a standard deviation of 9% and maximum deviation of 27.9% for spherical surfaces. For immersed tubes, the prediction of equation (20a,b) has a standard deviation of 16.7% and maximum deviation of 37.2%.

The analysis of particle movements in fluidized beds may also be used for the classification of fine particles to large particles, i.e. the Group B to Group D powder, as suggested by Geldart [31]. The maximum heat transfer rate is reached when $|\bar{u}_s| = \bar{u}_b$ for fine particles and $|\bar{u}_s| = u_0/\varepsilon$ for large particles. therefore, it is reasonable to classify these two groups by the relation

$$|\bar{u}_s| = \bar{u}_b = \frac{u_{mf}}{\varepsilon_{mf}}. \quad (23)$$

According to Ishida's results, when $|\bar{u}_s|$ approaches $\bar{u}_b$, $(u_{opt} - u_{mf})$ is about 0.5 $\bar{u}_b$, and assuming $\varepsilon_{mf} = 0.4$, equation (22) becomes

$$u_{opt} = 2.25\, u_{mf}. \quad (24)$$

As can be seen from Fig. 3 that is equivalent to $Ar = 2\times10^4$. Therefore, $Ar = 2\times10^4$ can be taken as the classification of fine particles to large particles. This was also observed by Chen and Pei [32]. In a fluidized bed of very large particles ($Ar > 10^7$), the particle mixing within the bed is less effective, the interstitial gas convective heat transfer dominates, the overall heat transfer coefficient is accounted for by the gas convective transfer component [33].

The above analysis is also useful for predicting the bed voidage at optimum gas flow-rate, e.g. for 2×10^4

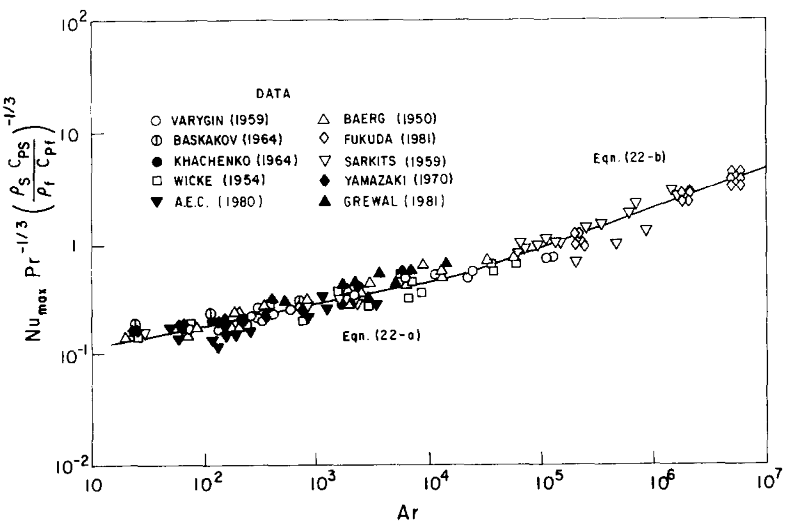

FIG. 5. Correlation of Nu_{max}, equation (22a,b).

$< Ar < 10^7$, u_{opt} is less than 2.25 u_{mf} and $|\bar{u}_s|$ is proportional to $(u_0 - u_{mf})^{1.4}$. Substituting this relation into equation (16) and comparing it with equation (20b), it is found that $(1-\varepsilon_{opt})$ is almost constant in this region.

However, for $Ar < 2\times10^4$, at optimum gas flow-rate, $|\bar{u}_s|$ might be proportional to $(u_0-u_{mf})^{0.7}$. Combining this relation with equation (16) and comparing it with equation (20a) it can be seen that $(1-\varepsilon_{opt})$ is proportional to $Ar^{0.18}$ or $d_p^{0.54}$, which is also observed by Yamazaki and Jimbo [3].

Moreover, equation (7) may also be applicable to non-optimum cases in fluidized bed heat transfer. As shown in Fig. 2 that, in ascending region, $|\bar{u}_s|$ seems to be proportional to $(u_0-u_{mf})^{1.4}$. Therefore, in fine particle systems, the heat transfer coefficient in ascending region may be correlated as

$$Nu/Nu_{max} = \text{const.}\, Ar^n (Re - Re_{mf})^{0.7}. \qquad (25)$$

5. CONCLUSIONS

(1) A new model of the heat transfer between fluidized beds and immersed surfaces has been developed based on the newly defined two-phase boundary layer and surface renewal theory.

(2) In considering the particle movements as an important factor for predicting the optimum flow-rate of the fluid, a general correlation is obtained.

(3) Based on the new model, a correlation for predicting the maximum heat transfer coefficient is also obtained. It correlates well with the literature data under various experimental conditions.

(4) According to the analysis on particle movements in the model. $Ar = 2\times10^4$ can be taken as the classification of fine particles to large particles, i.e. the Group B and Group D powder defined by Geldart [31].

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UN MODELE DE TRANSFERT THERMIQUE ENTRE DES LITS FLUIDISES ET DES SURFACES IMMERGEES

Résumé—Dans les lits fluidisés pleinement établis, le mouvement des particules joue un rôle important dans le mécanisme du transfert thermique. Considérant cela, un nouveau modèle est proposé pour décrire le transfert thermique entre des lits fluidisés et des surfaces immergées, à partir de la définition d'une couche limite diphasique autour de la surface. On obtient des formules du coefficient de transfert maximal et du débit optimal. Elles s'accordent bien avec les données connues dans un large domaine de conditions.

EIN MODELL DER WÄRMEÜBERTRAGUNG ZWISCHEN FLIESSBETTEN UND UMSPÜLTEN OBERFLÄCHEN

Zusammenfassung—In voll ausgebildeten Fließbetten spielt die Partikelbewegung eine überragend wichtige Rolle beim Wärmeübertragungsprozeß. Unter Berücksichtigung dieses Einflusses wurde ein neues Modell auf der Grundlage einer thermischen Zweiphasengrenzschicht an der Oberfläche entwickelt, das den Wärmeübergang zwischen Fließbetten und umspülten Oberflächen beschreibt. Daraus werden später Beziehungen für den maximalen Wärmeübergangs-Koeffizienten und für den optimalen Volumenstrom ermittelt. Sie stimmen für einen weiten Bereich von Bedingungen gut mit den in der Literatur angegebenen Daten überein.

МОДЕЛЬ ТЕПЛОПЕРЕНОСА МЕЖДУ ПСЕВДООЖИЖЕННЫМИ СЛОЯМИ И ПОГРУЖЕННЫМИ ПОВЕРХНОСТЯМИ

Аннотация—В полностью развитых псевдоожиженных слоях движение частиц оказывает существенное влияние на процессы теплопереноса. С учетом этого разработана новая модель теплопереноса между псевдоожиженными слоями и погруженными поверхностями, основанная на определении двухфазного теплового пограничного слоя вокруг поверхности. Выведены уравнения максимального коэффициента теплообмена и оптимальной скорости течения. В широком диапазоне условий они хорошо согласуются с литературными данными.